Conditional Probability

Finite Math

14 May 2019

Introductory Example

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In the experiments we were doing earlier that involved doing things multiple times, the outcome of the first thing did not influence the later outcomes. As we can see with the previous example, this is not the case for this experiment (which is known as "sampling without replacement"). This is the notion of *conditional probability*, denoted by $P(A|B)$ "the probability of A given B."

Definition

Definition (Conditional Probability)

For events A and B in an arbitrary sample space S, we define the conditional probability of A given B by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where $P(B) \neq 0$.

Now You Try It!

Example

Suppose we roll a single fair die (6 sided), but cannot see the outcome. What is the probability that we rolled a prime number given that someone has told us that we rolled an odd number?

Application Example

Example

Suppose that city records produced the following probability data on a driver being in an accident on the last day of a Memorial Day weekend:

	Accident (A)	**No Accident** (A')	**Totals**
Rain (R)	$.025$	$.335$	$.360$
No Rain (R')	$.015$	$.625$	$.640$
Totals	$.040$	$.960$	1.000

(a) *Find the probability of an accident, rain or no rain.*

(b) *Find the probability of rain, accident or no accident.*

(c) *Find the probability of an accident and rain.*

(d) *Find the probability of an accident, given rain.*

Probability of an Intersection

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$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad \text{and} \qquad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

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$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A).$$

We can use either product to compute the probability of the intersection.

Example

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If 40% of a department store's customers are male and 80% of the male customers have store credit cards, what is the probability that a customer selected at random is a male and has a store credit card?

Now You Try It!

Example

An automobile manufacturer produces 37% of its cars at plant A. If 5% of the cars manufactured at plant A have defective emission control devices, what is the probability that one of this manufacturer's cars was manufactured at plant A and has a defective emission control device?

Creating Probability Trees

We can create trees diagrams based on the number of outcomes in a sequence of experiments similar to how we used them for counting problems.

Example

Two balls are drawn in succession, without replacement, from a pot containing 3 *blue balls and* 2 *white balls. What is the probability of drawing a white ball on the second draw?*

Now You Try It!

Example

Two balls are drawn in succession without replacement from a box containing 4 *red and* 2 *white balls. What is the probability of drawing a red ball on the second draw?*

Comparison Example

Example

Consider again the experiment where two balls are drawn in succession, without replacement, from a pot containing 3 *blue balls and* 2 *white balls. What difference would it make if we changed "without replacement" to "with replacement"? Let A be the event that a white ball is pulled on the second draw and let B be the event that a white ball is pulled on the first draw. Compute $P(A|B)$ and $P(A)$ in both scenarios: without and with replacement. What do you notice?*

Independence

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If A and B are any events in a sample space S, we say that A and B are independent if

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A consequence of this definition is that if A and B are independent events, we have

$$P(A|B) = P(A)$$

and

$$P(B|A) = P(B).$$

Testing for Independence

Example

A single card is drawn from a standard 52-card deck. Test the following for independence:

(a)

$$\begin{aligned} E &= \textit{the drawn card is a spade} \\ F &= \textit{the drawn card is a face card} \end{aligned}$$

(b)

$$\begin{aligned} G &= \textit{the drawn card is a king} \\ H &= \textit{the drawn card is a queen} \end{aligned}$$